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On-off intermittency in random map lattices

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In this paper we study numerically an ensemble of random driving logistic maps. There are two accessible cases of on-off intermittency in the system for different values of parameters. The first one corresponds to the loss of stability of the fixed point, while the second one is due to the instability of the synchronous motion of the ensemble. It shows that the two cases of intermittency belong to the same class of universality.

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Complex systems, ranging from economic markets and ecosystems to earthquakes and turbulent fluids, have generated a lot of research interest in recent years. The most striking feature of many composite systems containing a large number of elements is that fascinating global phenomena arise out of seemingly simple local dynamics. It is then of considerable importance to investigate the physics generic to such spatially extend systems. Much more work [1–7] has been done on the so-called one-dimensional diffussively coupled map lattices (CML's) given by

$$y_{n+1}^{(i)}=(1-\epsilon)f(y_n^{(i)})+\frac{\epsilon}{2}[f(y_n^{(i+1)})+f(y_n^{(i-1)})], \quad (1)$$

where n is the discrete time step, i is the lattice point, and $f(y)$ prescribes the local dynamics at each lattice site. A different but related class of spatiotemporal model systems being considered is the following noise-driven uncoupled map lattices (UCML's):

$$y_{n+1}^{(i)}=F(y_n^{(i)},a_n^{(i)}), \quad (2)$$

where $a_n^{(i)}$ can be a random variable influncing the the dynamics at site i and

$$F(y_n^{(i)},0)=f(y_n^{(i)}). \quad (3)$$

This type of model may be motivated in part by considering a hypothetical physical situation in which a system consisting of L identical units is embedded in a noisy environment. If the coupling between units is suffícently weak, it can be neglected in the first approximation [8,9]. In our paper, the special case of $a_n^{(i)}=\xi_n$ will be studied, where ξ_n is a random variable. This corresponds to a random background that is homogeneous in space.

Recently, a type of intermittency behavior known as "on-off intermittency" has been reported [10–12]. This intermittency is the competition of two states. The "on" state is a short burst while the "off" state is a long period of constant state. A set of five coupled differential equations which can produce "on-off intermittency" was given by Platt, Sprigel, and Tresser [10]. And a simpler model of a one parameter random driving map was also studied in detail [11]. At the onset of intermittency behavior, the distribution of laminar phase exhibits a universal asymptotic $-3/2$ power law. The mean laminar phase displays a power law as a function of the coupling strength with a critical exponent -1 . On the other hand, Yu, Ott, and Chen [13] have studied a class of two-dimensional maps with randomly varying parameters. These maps exhibit so-called snapshot attractors. During the iteration, the size of the snapshot attractor can undergo a form of intermittency behavior that is similar to on-off intermittency. Heagy, Platt, and Hammel claimed that these two behaviors had some essential difference [11]. We found, however, that in a random driving UCML with local function of logistic map, both phenomena can be observed. The same scaling relations are found for both attractors. It seems that the two manifestation of intermittency belong to the same class of universality.

The maps we study are of the form

$$y_{n+1}^{(i)}=z_n f(y_n^{(i)}), \quad (4)$$

where $i=1, 2, \ldots, L$, labels the ith particle (i.e., initial condition), and

$$f(y)=y(1-y), \quad (5)$$

$$z_n=ax_n+b, \quad (6)$$

with x_n a random variable uniformly distributed in interval [0,1]. The inequalities

$$a>0, \quad b>0, \quad a+b<4 \quad (7)$$

should be satisfied because the iteration of Eq. (4) must be bounded. These conditions define a triangle in the a-b plane. A total number L of initial conditions in interval [0,1] with uniform distribution is taken. And the values of control parameter z_n at a certain step of iteration are identical for every initial condition. This means that all the initial conditions iterate with the same function.

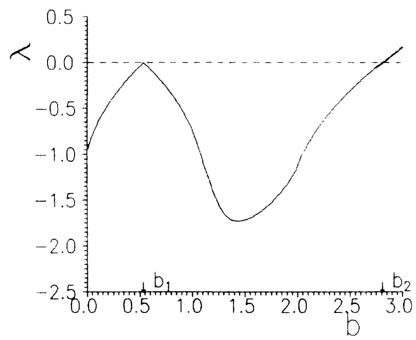

FIG. 1. The Lyapunov exponent for a single map with $a=1.0$.

Chaotical or periodic motion in a single map is characterized by a positive or negative Lyapunov exponent defined by

$$\lambda=\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\ln\left|\frac{dy_{n+1}}{dy_n}\right|. \tag{8}$$

For the independence of the sites in UCML's, the Lyapunov exponents are equal to each other for all the sites. We assume for the moment that $a=1.0$ in Eq. (6). Numerical calculation for a single map shows that (see Fig. 1) with the increasing of the value of b, the Lyapunov exponent λ increases gradually to zero, and then decreases to a minimum, but it will eventually increase beyond zero. So there are two zero points of Lyapunov exponents corresponding to two critical values $b_1\simeq0.54$ and $b_2\simeq2.82$. With different values of a we obtain two critical curves in the a-b plane which divide the triangle (7) into three regions (see Fig. 2). Numerical simulation shows that the states in each region are different from each other.

The first critical curve (with $b=b_1$) corresponds to the loss of stability of the fixed point $y_n=0$. It is a stable attractor when $b<b_1$, and becomes unstable when $b>b_1$. This curve is determined by the equation [11]

$$(b_1+1)\ln(b_1+1)-1-b_1\ln b_1=0. \tag{9}$$

This corresponds to the onset for "on-off intermittency" of signal as y_n. This phenomenon can be observed even for a single map as studied in Refs. [10–12]. However, for different initial conditions even with values of b beyond this critical point, the motion of the large number of particles in the system is always synchronous, i.e., $y_n^{(i)}=y_n$ for all i, though the motion of a single site is random. The negative Lyapunov exponent implies that after a long transient time all the particles must clump at a single point. From the analytical study in Ref. [11], the probability of laminar phases with length n shows a power-law distribution with an exponential decay at large size:

$$\Lambda_y(n)\propto n^{-3/2}\exp(-n/n_y), \tag{10}$$

where

$$n_y\propto(b-b_1)^{-2}. \tag{11}$$

The mean laminar phases act as

$$\langle n\rangle\propto(b-b_1)^{-1}. \tag{12}$$

In addition, we find in numerical calculation that the mean amplitude of signal y_n

$$\langle y\rangle\equiv\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}y_n\propto(b-b_1). \tag{13}$$

The numerical study for $a=1$ (see Fig. 3), shows that $\langle y\rangle=0.92(b-0.54)$. From this, the critical value of b is obtained as $b_1=0.54$, in agreement with the first critical point in Fig. 1.

Near the other critical curve $b=b_2$, the behavior of the particle distribution is similar to that in Ref. [13]. For example, with $a=1$ and $b=2.82$, we may obtain a snapshot attractor by sprinkling a large number of initial points uniformly in the interval [0,1], then iterating each point under the map (4) for a large number of iterations. The size of the snapshot attractor s_n at time n is defined the same as in Ref. [12]:

$$s_n=\left[\frac{1}{L}\sum_{i=1}^{L}(y_n^{(i)}-\bar{y}_n)^2\right]^{1/2}, \tag{14}$$

where $\bar{y}_n$ is the spatial average of $y_n^{(i)}$:

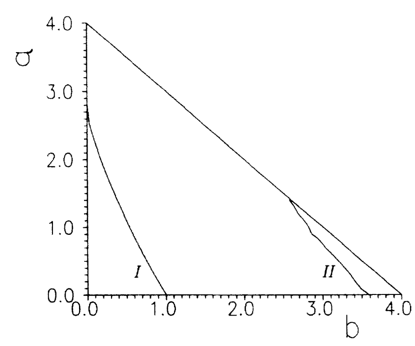

FIG. 2. The phase diagram in the a-b plane. Line I is the onset condition for the "on-off intermittency" of signal as y_n, and line II as s_n.

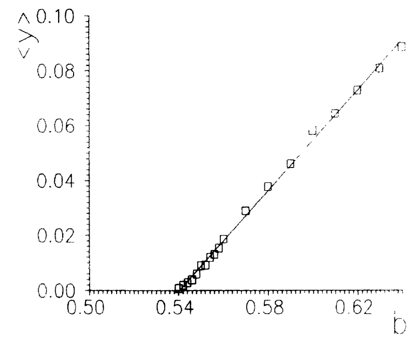

FIG. 3. The mean amplitude of signal y_n vs the variation from the critical line $(b-b_1)$, with $a=1.0$. The least-square fit is $\langle y\rangle=0.92(b-0.54)$.

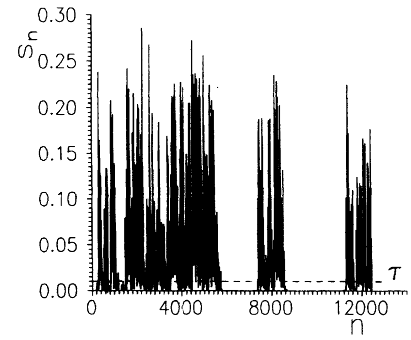

FIG. 4. The size of the snapshot attractor s_n vs the iterate steps n on a linear scale with $a=1.0$, $b=2.82$. Below the threshold $\tau=10^{-2}$ the signal s_n is considered to be in laminar phase. The size of the ensemble used is $L=200$.

$$\bar{y}_n=\frac{1}{L}\sum_{i=1}^{L} y_n^{(i)}. \tag{15}$$

Just beyond the critical value b_2, the intermittency of the signal as s_n was observed. Figure 4 shows the intermittency of s_n on a linear scale. It can be seen that time intervals where s_n is extremely small are punctuated by a shorter burst where s_n is of order 1. As b increases from b_2, the intervals of time with s_n of order 1 become more and more dense. We may also call this intermittency "on-off intermittency," because it follows the same scaling relations as those found at the first critical curve. Since near the second critical line, the Lyapunov exponent λ is a linear function of b, i.e., $\lambda=\beta(b-b_2)$, so it could also be used as the control parameter. Near the critical curve $\lambda=0$ (i.e., $b=b_2$) the laminar phases have an asymptotic power-law distribution with an exponential decay at large size:

$$\Lambda_s(n)\propto n^{-3/2}\exp(-n/n_s), \tag{16}$$

with

$$n_s\propto\lambda^{-2}. \tag{17}$$

Figure 5 shows the plot of $\Lambda_s(n)$ from numerical simulation

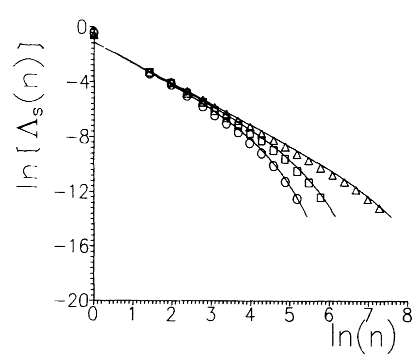

FIG. 5. The numerical results for the distribution $\Lambda_s(n)$ of laminar phases with $L=20$, $a=1.0$, (△) $b=2.84$, (□) $b=2.90$, and (○) $b=2.96$. They are best fitted by Eq. (15) with the cutoffs n_s respectively 1450, 135, and 50, shown as the solid curves.

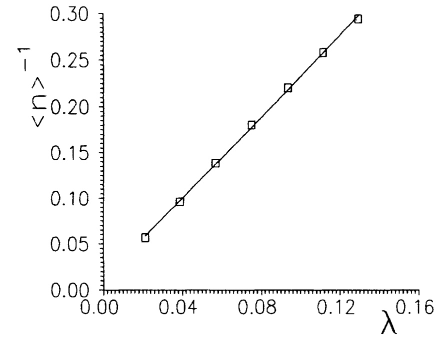

FIG. 6. The mean length $\langle n\rangle$ of laminar phases vs λ with $a=1.0$.

of maps (4) with $a=1.0$. A total of 10^6 intervals of laminar phase were used to construct the distribution. The threshold from a laminar phase is fixed at $\tau=10^{-2}$. Since the value of n_s is usually quite large (about 10^2-10^3), it is not easy to confirm numerically the scaling relation (16). For the mean laminar phases $\langle n\rangle$, however, we find from Eqs. (15) and (16) that it should be of the form

$$\langle n\rangle=\sum_n n\Lambda_s(n)\propto n_s^{1/2}\propto\lambda^{-1}, \tag{18}$$

which is easier to confirm in numerical simulation. The least-square fit of data in Fig. 6 shows that $\langle n\rangle \propto 1/\lambda$. In addition, the mean amplitude of signal s_n is $\langle s\rangle \propto \lambda$. This is similar to that in Ref. [13].

From the above numerical calculation, we conclude that these two manifestations of intermittency have all the same critical exponents. The intermittency of signal y_n is caused by the instability of the fixed point $y=0$, while that of signal s_n is due to the instability of the orbit y_n. It is known that the first case can be mapped into a random or chaotic walk problem for random or chaotic driving. The step length is $r_n=\ln z_n$ and the sign of $\langle r_n\rangle$ determines the stabilities of the fixed point $y=0$ of iteration (4). The distribution of laminar phases and other results can be calculated analytically. Much more detail can be found in Ref. [11]. For the second case it is not easy to map the motion of s_n into a random or chaotic walk. However, the same scalling behavior might imply that there must be some connection between these two problems. We also presume that on-off intermittency might be observed near every zero point of the Lyapunov exponent for many cases.

It should also be pointed out that some other interesting things are expected near the b axis (i.e., as $a\to 0$). Just on the b axis, there is a cascade of period doubling bifurcations as well as many period windows. They disappear completely for the strong influence of random driving, though at small values of a these bifurcations can still be observed. The crossover of these two situations will be studied elsewhere.

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